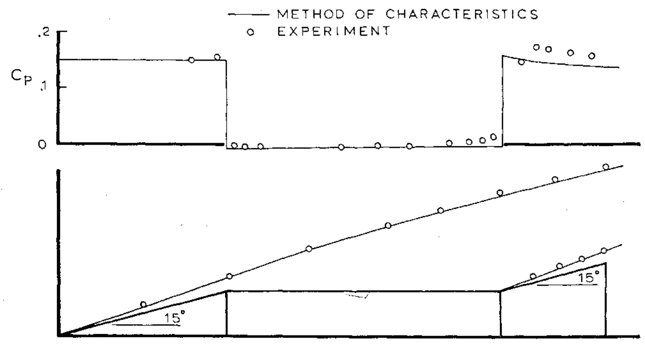

Fig. 1 Pressure distribution and shock shapes on the sharp nosed body, $r_n/r_b = 0$.

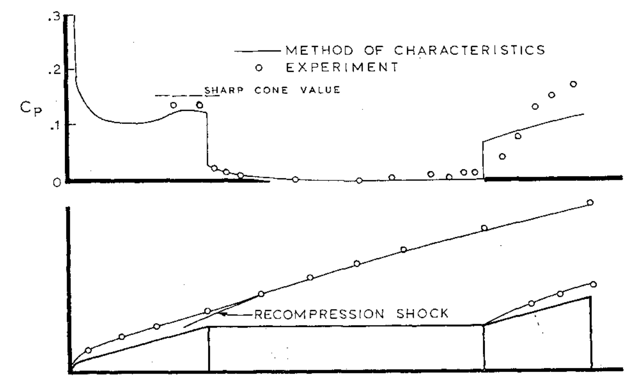

Fig. 2 Pressure distribution and shock shapes on the slightly blunted body, $r_n/r_b = 0.2273$.

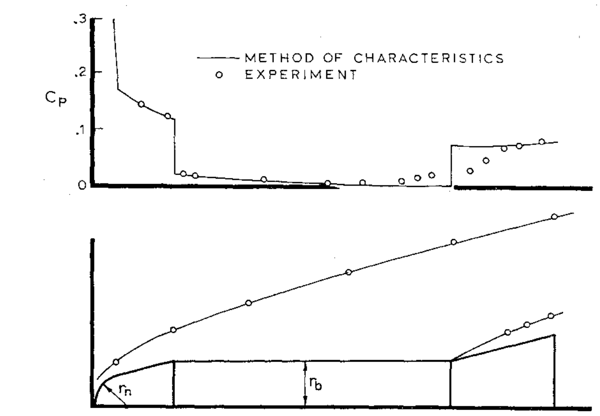

Fig. 3 Pressure distribution and shock shapes on the moderately blunted body, $r_n/r_b = 0.6825$.

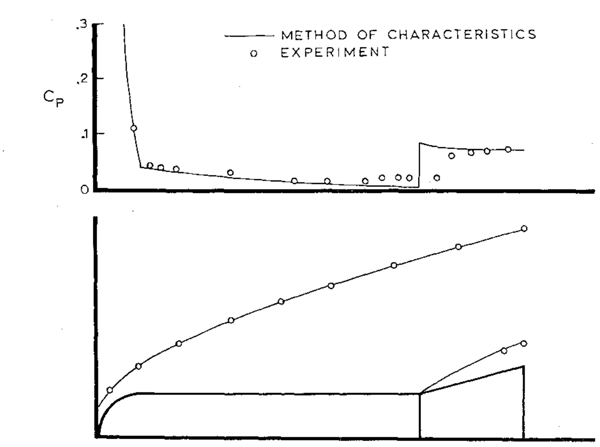

Fig. 4 Pressure distribution and shock shapes on the completely blunted body, $r_n/r_b = 1.0$.

pressure at the rear of the flare in Fig. 3 has started to increase to the sharp nose value.

The figures presented also illustrate the effect of separation on the pressure near the flare juncture. Figure 2 shows that separation can cause pressures on the flare to be considerably higher than the theoretical values. Separation was indicated clearly in the schlieren photographs, and also the flare shock waves were not attached to the body. A thorough study of separation is currently being conducted and, therefore, is not treated here.

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Penetration of Spacecraft by Lunar Secondary Meteoroids

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It is recognized that the flux of meteoroids in cislunar space is sufficiently large that it must be taken into account in designing the protective skin of a spacecraft.[1] Recent studies at the Ames Laboratory reported by Gault, Shoemaker, and Moore[2] on hypervelocity impact processes have shown that the total mass ejected by a hypervelocity particle may exceed the mass of the incident particle by several orders of magnitude. This has raised the question as to whether the secondary meteoroids generated at the lunar surface might not appreciably increase the probability of puncture of a spacecraft when it is exposed to this secondary flux. It should be noted that 1) in traversing the ejecta cloud, the spacecraft itself may determine the mean impact velocity, and 2) the question of a "captured flux," or meteoroid concentration near the moon, is not settled. The purpose of this note is to examine, on rather general grounds, the problem of the spacecraft at rest on the lunar surface and to set limits on the enhanced probability of puncture.

The basic argument is that the total kinetic energy available to be distributed to the secondary particles is bounded by the kinetic energy of the primary particle. This allows us to write down rather powerful restrictions on the integrated energy spectrum of the secondary particles and, in particular, to derive the increase in the flux of particles exceeding a given energy. The assumption is made that the thickness of protective skin penetrated by a particle depends only on the kinetic energy,[1-3] and hence it is shown that the probability of puncture is, at most, increased by a small factor. We now proceed with the argument.

Kinetic energy is, at most, conserved in a primary impact. The secondary particles, whatever their combined mass, share the incident energy. This process could double the energy flux incident on the moon. The arguments in this paper require that there be few tertiary particles capable of puncturing a spacecraft, i.e., that the energy flux cannot be more than doubled. The velocities of the secondaries must be quite small. First, the mass-multiplication in the impact is very large; to balance this, the mean velocities must be

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low. Second, the few particles having velocities exceeding 2.4 km/sec will leave the moon. The great number of secondary impacts will be subsonic and are unlikely to generate penetrating tertiary particles. The energy flux is then, at most, doubled.

The assumption that penetration depends only on kinetic energy sets for a given spacecraft wall a critical energy for penetration E_c. We are then concerned with the total particle flux having energies exceeding E_c; this will be a sum of the primary flux, $N_p(E_c)$, and a secondary flux, $N_s(E_c)$. The total flux is

$$N_p(E_c) + N_s(E_c) = N_p(E_c)[1 + K(E_c)] \qquad (1)$$

where K, the multiplication factor in the primary impact process, may be a function of E_c:

$$N_s(E_c) = K(E_c)N_p(E_c) \qquad (2)$$

The law of the conservation of energy now is applied to obtain a limit on K. To compute the energy content of the secondary flux, let

$$-N_s'(E) = (d/dE)N_s(E) \qquad (3)$$

Then,

$$N_s(E_c) = \int_{E_c}^{U_s} N_s'(E)dE \qquad (4)$$

where U_s is an upper bound on the energy of a secondary particle. The energy flux of penetrating secondaries, $W_s(E_c)$, is then

$$W_s(E_c) = \int_{E_c}^{U_s} E\cdot N_s'(E)dE \qquad (5)$$

The primary energy flux, $W_p(E_c)$, is written similarly as

$$W_p(E_c) = \int_{E_c}^{U_p} E\cdot N_p'(E)dE \qquad (6)$$

where U_p is an upper bound on the energy of a primary particle. The bounds U_p and U_s may in most cases be considered infinite. The conservation of energy assures that

$$W_s(E_c) < W_p(E_c) \qquad (7)$$

The inequality in (7) is considerable, since the primary of energy E_c generates particles of energy less than E_c. $W_s(E_c)$ is only a part of the energy contained in the secondaries. To obtain a value for the multiplication $K(E_c)$, Eqs. (2) and (7) are combined to find

$$K(E_c) < \frac{W_p(E_c)/N_p(E_c)}{W_s(E_c)/N_s(E_c)} = \frac{\overline{E_p}}{\overline{E_s}} \qquad (8)$$

Here, $\overline{E_p}$ and $\overline{E_s}$ are the average particle energies over the flux range above E_c, i.e.,

$$\overline{E_p} = \frac{1}{N_p(E_c)} \int_{E_c}^{U_p} E\cdot N_p'(E)dE \qquad (9)$$

The average energy is determined by the shape of the energy spectrum. If the primary and secondary spectra are similar, then the number of penetrating particles scales with the energy flux and is, at most, doubled.

This is considered a reasonable case, and the result holds regardless of the primary spectrum. A commonly assumed spectrum (for either primary or secondary) has the form

$$N(E) = \text{const } E^{-s} \qquad (10)$$

Using Eqs. (10) and (3), Eq. (9) is evaluated for various values of s, and Table 1 is obtained. On the first line, a distribution applicable to the secondaries is considered; this is a "worst" case in which all particles have the single energy E_c.

Upper bounds now are computed on the multiplication factor $K(E_c)$ for several special distributions. The data

Table 1 The average energy of particles of $E \geq E_c$ for various distributions

Energy distribution	Average energy
Mono-energetic particles, energy E_c	E_c
$s > 1$	$E_c[s/(s-1)]$
$s = 1$	$E_c \cdot \ln U/E_c$

Table 2 Maximum multiplication factor $K = \overline{E_p}/\overline{E_s}$

Primary spectrum	Secondary spectrum $E = E_c$	$S = 1.34$	$s = 1$
$E = E_c$	1	0.26	0.043
$s = 1.34$	3.9	1	0.17
$s = 1$	27.6	7	1.2

from the Harvard photographic meteor project suggest that $s = 1.34$.[4] Past workers (first, Watson[5]) have used the convenient form $s = 1$. For the special case $s = 1$, the average energy $\overline{E} \to \infty$ as $U \to \infty$. In order to remove this divergence, we choose the upper limit U_p by asserting that, for a given mission time, we are not concerned with primary impacts on the moon having a probability of occurrence smaller than the probability of impact of a penetrating primary on the spacecraft. From the cumulative distribution function, we find U_p/E_c = area of moon/area of spacecraft $\simeq 10^{12}$. We assume arbitrarily that the bound on the secondaries $U_s = 10^{-2}U_p$.

Table 2 shows multiplication factors K for various primary and secondary spectra. For illustrative purposes, the numerical values chosen for s are 1.34, 1, and the mono-energetic distribution.

It will be noted that, even for the Watson distribution ($s = 1$), the maximum multiplication is under 30 times in the worst case of mono-energetic secondaries.

It should be emphasized that the multiplication factors in the lower left in Table 2 are considerable overestimates because 1) conservation of kinetic energy has been assumed during the impact process; 2) a secondary spectrum markedly different from the primary spectrum has been assumed before reaching the factor of 27.6; and 3) the mono-energetic spectrum for the secondaries has been fixed arbitrarily at the critical energy for penetration.

A conservative experimental primary flux distribution is well represented by relation (10) with $s = 1$ or steeper, over a range of 10^{12}. A reasonable "penetrating flux" for the spacecraft might be $10^{-10}m^{-2}sec^{-1}$, assuming an exposure of 30 m^2 for three or four days and a 0.999 probability of no puncture. A flux 10^{-12} of this lies in Brown's meteorite population.[6] Though the intervening data show a variety of slopes, the overall behavior is well represented by $s = 1$ or steeper.

Finally, there is the question of the energy-sensitive penetration law. Laboratory experiments support this in the velocity range below 10 km/sec, but the extrapolation to primary meteoroid velocities (as high as 70 km/sec) is in question. Penetration may be momentum-sensitive in that range. However, this has no influence on the multiplication of the energy flux or on the hazard from secondary particles, whose velocities are low.

It can be concluded, therefore, that within the assumptions given in the foregoing, the probability of penetration of a spacecraft will not be appreciably larger on the lunar surface than it is in deep space.

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Viscoelastic Cylinders of Complex Cross Section under Axial Acceleration Loads

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A KNOWLEDGE of the stresses and deformations in solid propellant rocket motors due to axial acceleration loads is necessary for analysis of motor structural integrity. This note deals with the axial acceleration of propellant grains of infinite length whose internal perforations are not circular but have a number of axes of symmetry, such as the common types of star perforations.

Equations of Motion

The equations of motion for a viscoelastic solid may be shown to be of the form

$$\left(K + \frac{G}{3}\right)\left[\frac{\partial e(t)}{\partial x}\right] + G\,[\nabla^2 u(t)] + X = \gamma \frac{\partial^2 u}{\partial t^2} \qquad (1)$$

with permutations on the coordinates x,y,z, the displacements u,v,w, and the body forces X,Y,Z. The functional notation

$$G[f(t)] = f(t)G(0) + \int_0^t f(t_1)\frac{d}{dt_1}\,[G(t - t_1)]dt_1 \qquad (2)$$

is used. Here $G(t)$ is the viscoelastic shear relaxation modulus, $K(t)$ is the viscoelastic bulk relaxation modulus, γ is the material density, t is time, ∇^2 is the Laplacian operator, and e is the dilatation. This formulation, with the exception of acceleration and body force terms, has been given by Elder.[1]

Under the restriction that u, v, and w are invariant with z, the first two of the equations indicated by Eq. (1) reduce to the usual plane strain equations of linear viscoelasticity which do not contain the displacement w. The third equation which is now uncoupled from the first two, is

$$G\left[\frac{\partial^2 w(t)}{\partial x^2} + \frac{\partial^2 w(t)}{\partial y^2}\right] + Z = \gamma \frac{\partial^2 w}{\partial t^2} \qquad (3)$$

Henceforth, consideration will be given to the solution of Eq. (3) with geometries limited to cylinders having generators parallel to the z axis and body forces consisting only of the weight of the body. It is evident that the weight per unit volume can be expressed as the product of the density γ and a pseudo acceleration, the gravitational constant, and can thus be included in the right side of Eq. (3). Consequently, the body force will no longer be explicitly considered.

Boundary conditions applicable to Eq. (3) may consist of the specification of the displacement or shear stress as prescribed functions of time. Typically, these may take the form

$$w(t) = f(t) \text{ on } B_1 \qquad (4)$$

and

$$\tau_{nz}(t) = h(t) \text{ on } B_2 \qquad (5)$$

where B_1 and B_2 are the boundaries of a hollow cylindrical body, and n denotes the outward normal to the surface.

The displacement of any point w can be considered to consist of two parts (w_1 and w_2), where w_1, a function of time only, is the displacement of boundary B_1 and w_2 is the displacement of the point relative to the boundary B_1. The displacement w_1 may be associated with a rigid body displacement. The displacement w_2 is associated with the deformation of the cylinder material and is a function of the space coordinates x and y and of time t. The displacement w_2 will be further restricted by neglecting dynamic effects so that the magnitude of the variation of w_2 with time is much less than that of w_1. Thus,

$$\partial^2 w/\partial t^2 \approx \partial^2 w_1/\partial t^2 \equiv A(t)$$

and since

$$\partial w_1/\partial x = \partial w_1/\partial y = 0$$

Eq. (3) may be written as

$$G\left[\frac{\partial^2 w_2(t)}{\partial x^2} + \frac{\partial^2 w_2(t)}{\partial y^2}\right] = \gamma A(t) \qquad (6)$$

where $A(t)$ is a specified acceleration [$A(t) = 0, t < 0$]. Specification of the over-all body acceleration may now be made independently of the displacements associated with deformations.

To obtain a solution of Eq. (6), it is convenient to use the Laplace transform of the equation with respect to time and to replace the transformed stress relaxation modulus $\bar{G}(s)$ in the resulting expression by the transformed creep compliance $\bar{J}(s)$ by using the relation

$$s\bar{G}(s) = [s\bar{J}(s)]^{-1}$$

Performing these operations and taking the inverse Laplace transform, results in

$$\nabla^2 L^{-1}\{\bar{w}_2(s)\} = L^{-1}\{\gamma s\bar{A}(s)\bar{J}(s)\} \qquad (7)$$

where $L^{-1}\{\ \}$ denotes the inverse Laplace transform. If a displacement function ψ is defined as

$$\psi = \frac{L^{-1}\{\bar{w}_2(s)\}}{L^{-1}\{\gamma s\bar{J}(s)\bar{A}(s)\}} \qquad (8)$$

Eq. (7) becomes

$$(\partial^2\psi/\partial x^2) + (\partial^2\psi/\partial y^2) = 1 \qquad (9)$$

which is valid regardless of the time dependence of the acceleration and regardless of the time dependence of the shear

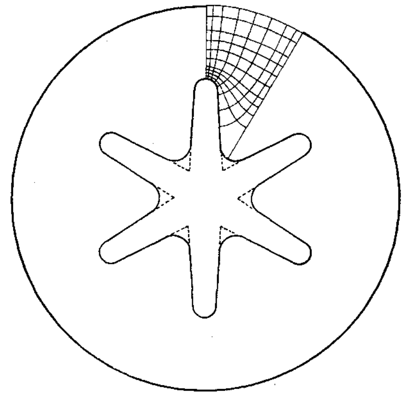

Fig. 1 Curvilinear co-ordinates obtained by conformal mapping.

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